INVESTIGATION OF THE CREEP OF THIN-WALLED SHELLS UNDER NONSTATIONARY LOADING

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There are creep investigations of structure elements under stationary loading in the literature [1, 4]. However, many elements of machines and installations in the creep state are exploited under conditions of nonstationary loading processes. The creep of structures under such conditions has been studied inadequately, and hence such investigations are quite urgent.

1. Creep under nonstationary loading is a complex phenomenon accompanied by many interesting effects. Experimental investigations of creep under complex loading indicate the essentially anisotropic nature of the hardening under creep. Classical theories of creep with isotropic hardening do not describe the strain anisotropy observable experimentally under complex loading programs. Hence, a number of authors proposed several modifications of the theory that take account of the effect of anisotropic hardening of materials. The theory developed by Malinin and Khazhinskii [3] most completely satisfies the requirements of adequacy and simplicity. Comparison of the experimental data with results obtained by this theory shows that this theory describes well the creep under step loading, reflects the reverse creep phenomenon, and agrees well with the experimental investigation of stress relaxation. The theory is applicable to processes of strain over rectilinear, or almost rectilinear trajectories.

The fundamental statements of the theory under consideration reduce to the following. The material is assumed initially isotropic and incompressible under creep. Taking account of the strain anisotropy is performed by separating the stress tensor σ_{ij} into active S_{ij} and additional ρ_{ij} stress tensors

$$\sigma_{ij} = S_{ij} + \rho_{ij}. \tag{1.1}$$

The validity of an analogous equality for the total σ'_{ij}, active S'_{ij}, and additional ρ'_{ij} stress deviators is assumed

$$\sigma'_{ij} = S'_{ij} + \rho'_{ij}. \tag{1.2}$$

The equations of state of the material are written thus:

$$\dot{\varepsilon}^c_{ij} = \frac{3}{2} A \frac{Q(S_i)}{S_i} S'_{ij}. \tag{1.3}$$

They are examined jointly with the kinetic equations

$$\dot{\rho}'_{ij} = V(\sigma_i)\dot{\varepsilon}^c_{ij} - B\frac{Q(\rho_i)}{\rho_i}\rho'_{ij}. \tag{1.4}$$

Here $\dot{\varepsilon}^c_{ij}$ in the relationships (1.3) and (1.4) are the creep strain rate tensor components, σ_i, S_i, ρ_i are the intensities of the total, active, and additional stresses, respectively; A, B are certain material constants, the specific form of the functions Q, V is determined by the condition of best fit of the simple aftereffect curves, and the dot above a symbol denotes the derivative with respect to time.

The physical relationships (1.3) and (1.4) of the theory were used in an analysis of the creep of different structure elements, for instance, turbine disks [5], with strain anisotropy taken into account. Certain results of investigations of the creep of thin shells of hardening materials under nonstationary loading are discussed in this paper.

2. Let us consider a thin shell of revolution of arbitrary type. We refer it to a curvilinear, orthogonal α, β, z coordinate system, where α = const, β = const are the lines of principal curvature on the shell middle surface, and the coordinate z is measured along the normal to this surface.

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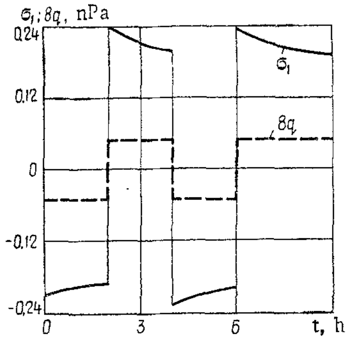

Fig. 1

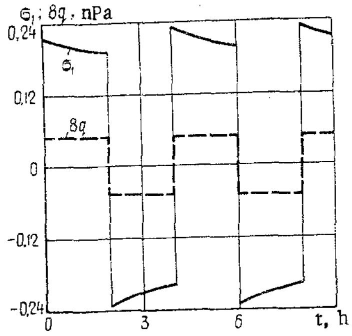

Fig. 2

The creep boundary value problem for thin-walled shells under axisymmetric loading can be represented [2] by a system of first order nonlinear differential equations

$$\dot{y}_i' = f_i(\dot{y}, \alpha) + f_{i*}(\dot{y}, y, \alpha) + b_i(\alpha) \qquad (i = \overline{1,6}) \tag{2.1}$$

with appropriate boundary conditions on the endfaces α_0, α_1

$$\dot{y}_k^j \cdot \Delta_{k+3j} + \dot{y}_{k+3}^j (1 - \Delta_{k+3j}) = 0; \quad k = \overline{1,3}; \quad j = \begin{cases} 0 & (\alpha = \alpha_0); \\ 1 & (\alpha = \alpha_1). \end{cases} \tag{2.2}$$

Here $y = \{y_i\}_{i=1}^{6} = \{T_1, Q_1, M_1, u, w, \theta\}^T$ is a vector comprised of force and geometric shell middle surface parameter, $\Delta = \{\Delta_i\}_{i=1}^{6}$ is an array of zeros and ones governing the type of boundary conditions, the dot above the functions denotes differentiation with respect to the time t, and the prime with respect to the meridian coordinate α.

The components of the vector f are linear functions of $\dot{y}$, and the vector b includes components of the rate of change in the surface loads. The components of f_* are implicit functional dependences corresponding to the physical nonlinearity of the problem. They are defined as linear combinations of certain integrals over the shell thickness [2], where the integrands are functions of the creep strain rate. Since the relationships (1.1)-(1.4) relate these latter to the stresses, the components of the vector f_* are therefore certain functionals of the stresses which are the unknowns.

Assuming the total strains ε_{ij} to consist of elasticity and creep strains, we have

$$\frac{d\sigma_{11}}{dt} = E_*[\dot{\varepsilon}_{11} - \dot{\varepsilon}_{11}^c + \nu(\dot{\varepsilon}_{22} - \dot{\varepsilon}_{22}^c)] \qquad (1 \rightleftarrows 2); \quad E_* = E/(1-\nu^2), \tag{2.3}$$

where E, ν are the elastic modulus and Poisson ratio of the material.

The relationships (2.3) are not integrable and cannot be solved for the stresses. These dependences can be rewritten in the form

$$\frac{d\sigma_{11}}{dt} = \Omega_1[\sigma_{11}(\alpha, z), \sigma_{22}(\alpha, z), \dot{y}(\alpha), z] \qquad (1 \rightleftarrows 2), \tag{2.4}$$

where Ω_i ($i = 1, 2$) are certain known functionals.

The solution of the boundary-value problem (2.1) and (2.2) is used in the operation to evaluate the functionals.

Therefore, the creep problem cannot be formulated in the standard sense. To determine the state of stress in the shell at some time, the initial problem for (2.4) and the boundary-value problem for (2.1) and (2.2) must be solved jointly. This mathematical singularity of the problem under consideration is due to its physical nonlinearity. The initial conditions for (2.4) are determined by the solution of the linear boundary-value problem of elastic strain of the shell.

Step methods in the time and coordinate are usually used to solve the problem formulated. The traditional step-by-step method of computation from stage to stage corresponds to the numerical solution of the Cauchy problem for (2.4) by using the Euler method in the argument t. Application of this method is related to the single solution of the linearized boundary value problem (2.1) and (2.2) in each step and to the

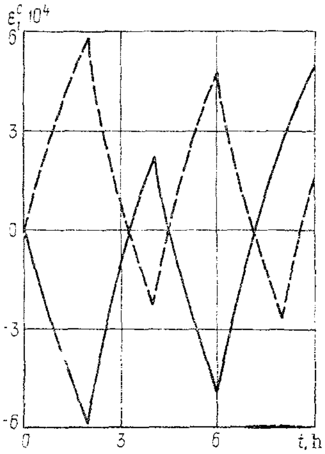

Fig. 3

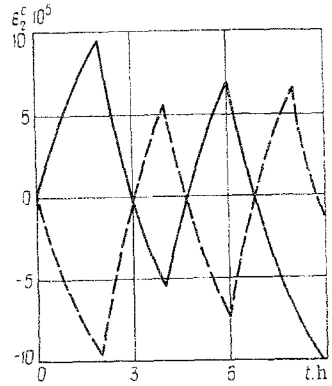

Fig. 4

inevitability of cumulative error as the steps are traversed. Hence, the utilization of more accurate methods of solving the initial problem than the Euler method is justified.

In this paper the Cauchy problem is integrated for the dependences (2.4) by using the Kutta–Merson method requiring the quintuple solution of the linearized boundary-value problem (2.1) and (2.2). However, the step method modification under consideration permitted increasing the step and the total medium of the calculations was thereby reduced. The magnitude of the time step was selected automatically from the condition of the error being accumulated in the computation not exceeding a certain given value. The integrals over the shell thickness were evaluated by the Simpson method. The linear boundary value problem was solved numerically by the Godunov stable method of discrete orthogonalization.

An algorithm was developed and realized on the BESM-6 electronic computer on the basis of the approach elucidated.

3. As an illustration, let us investigate the creep of a thin-walled cylinder clamped rigidly by one edge and hinge-supported by the other.

The geometric dimensions of the shell in meters are: length $l = 0.4$, thickness $h = 10^{-2}$, radius of the middle surface $R = 0.2$.

The loading conforms to uniform pressure q. Two cylinder loading programs are considered (Figs. 1 and 2). In both cases the load varied over a rectangular symmetric cycle for $t_* = 8$ h, and became constant after $t_* = 8$ h. The distinction is that in one case internal pressure acts initially on the shell during a half-cycle, and external pressure in the other case. The structure loading program displayed in Fig. 1 is ascribed the number 1, and the program in Fig. 2 the number 2.

The cylinder material is the alloy D16T at T = 473°K. The elasticity constants are E = 56 GPa and $\nu = 0.35$. Description of the curves of simple material aftereffect by using the relationships (1.1)-(1.4) permitted setting up the form of the functions in these equations and determining the values of the material constants

$$Q(S_i) = \operatorname{sh}\left(\frac{S_i}{F}\right); \qquad V(\sigma_i) = (C + D\sigma_i)^{-1};$$

$A = 1.32 \cdot 10^{-5}\ h^{-1}$, $F = 41.8$ MPa, $C = 7.3\ pPa^{-1}$, $D = 110\ nPa^{-2}$, $B = 0.16\ MPa \cdot h^{-1}$.

Eleven points of orthogonalization on the shell generator were used in the computations. The number of integration steps between adjacent points of orthogonalization was five. The cylinder thickness was divided into six equal sections. The "instantaneous" change in the external load was considered to occur after $\delta = 10^{-8}$ h. This time segment was used as a constant step in the integration of the initial problem for (2.4). In all the remaining cases the magnitude of the time step was selected automatically, where the accuracy in solving the Cauchy problem was 5% in the stresses.

Figures 1 and 2 illustrate the relaxation of the meridian stress on the shell inner surface in the fixing for the loading programs represented here. An insignificant stress redistribution in the half-cycle interval and a jump change for an instantaneous change in the external load are characteristic for this problem. As regards the creep strains in the structure, they grow rapidly with the lapse of time. Thus, the strain

$\varepsilon_1{}^c$ to the time $t_1 = 1$ h is 71% of the total strain ε_1 in an external fiber in the cylinder framing loaded according to the program 1.

The change in creep strain on the shell inner surface in a fixing is shown with the lapse of time in Figs. 3 and 4, where the solid line corresponds to the loading program 1, and the dashes to program 2. The influence of the loading history on the structure state of strain is noticeable.

As is seen from Fig. 3, the creep strain $\varepsilon_1{}^c$ in a shell loaded according to program 2 is at the time $t_2 =$ 1 h more than twice its value at the time $t_3 = 9$ h. Therefore, in this case cyclic loading hardens the material, which has the appearance of strain anisotropy.

Therefore, the results presented indicate the necessity to take account of strain anisotropy in creep problems for thin-walled shells from hardening materials. The method developed to solve such problems can be utilized in the analysis of the creep of the shell construction of different machines and power plants under nonstationary loading conditions.

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INVESTIGATION OF THE STABILITY OF COMPRESSED CYLINDRICAL SHELLS WITH THIN ECCENTRIC RIBS WITH A POLYNOMIAL APPROXIMATION OF DEFLECTIONS

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1. The solution for stringer shells, which is similar to that proposed but written more complexly and taking only radial interaction between the ribs and the skin into account, was obtained [7] for the general case of rib strain. The compatibility of the rib and skin strain in the radial and tangential directions is taken into account in [6], where the strain in the latter is represented by the tangential bending of the stringers. The general and particular cases of stringer shell strain are investigated in [2] with a discrete stringer arrangement and their eccentricity taken into account. The critical stresses are determined as the minimal roots of transcendental equations. The bending of the stringers in the tangential direction rather than their torsional-bending strain is also examined here. The thin-walled nature of the reinforcement is not taken into account.

The influence of a polynomial approximation of the deflections on the magnitude of the critical stresses of axial compression of stringer shells is investigated numerically [5]. The rib eccentricity is not taken into account. Thin-walled ribs were not considered. The least roots of the determinants were found by using the numerical solution.

It is shown [3] that the general case of stringer shell strain can be represented by short longitudinal and circumferential waves. Hence, taking into account the discrete location of the ribs is necessary even in this case. This deduction, expanding and supplementing the results of investigation [8], can be verified by the analysis of the characteristic equation (3.4) of this paper.

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